Problem 1: (15 points)

An object, mass m, moves so that its position is given by

$$x(t) = c_1 t^2 + c_2 t$$
$$y(t) = c_3 t + c_4$$

Here c_1, c_2, c_3 and c_4 are known constants.

a) Find the total force acting on the object.

$$v_x = \frac{dx}{dt} = 2c_1 t + c_2$$

$$v_y = \frac{dy}{dt} = c_3$$

$$a_x = \frac{dv_x}{dt} = 2c_1$$

$$a_y = 0$$

$$\boxed{\vec{F} = 2c_1 m \vec{i} + 0 \vec{j}}$$

b) Find the potential function for this force.

$$F_x = -\frac{\partial U}{\partial x}$$

$$U = -\int F_x \, dx = -\int 2c_1 m \, dx = -2c_1 m x + \text{Const}$$

$$F_x = -\frac{dU}{dx} = 2c_1 m$$

$$F_y = -\frac{dU}{dy} = 0$$

Problem 2: (15 points)

Two cars, one a compact with mass *m* and the other a large truck of mass 3*m*, collide head-on at typical freeway speeds.

a) Which car will experience a greater force as a result of collision? Explain your answer.

$\boxed{\text{same}}$ (Newton's third law)

b) Which car has a greater magnitude of momentum change? Explain your answer.

$\boxed{\text{same}}$ $p_{1x}^{before} + p_{2x}^{before} = p_{1x}^{after} + p_{2x}^{after}$

$|p_{1x}^{before} - p_{1x}^{after}| = |p_{2x}^{after} - p_{2x}^{before}|$

c) If the larger car changes its velocity by Δv, calculate the change in the velocity of the smaller car in terms of Δv.

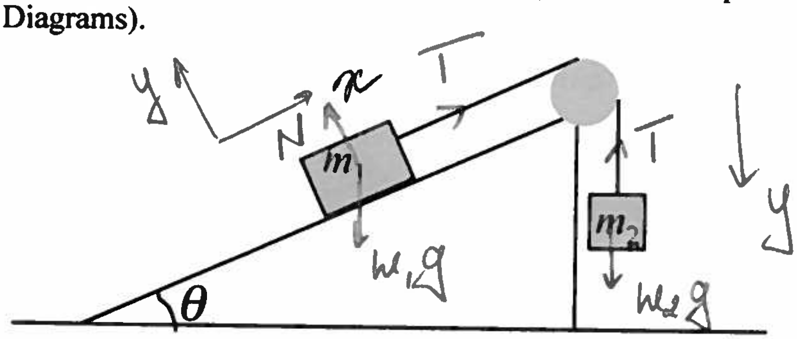
$$m v_{1x} + 3m v_{2x} = m u_{1x} + 3m u_{2x}$$

$$u_{1x} - v_{1x} = 3(v_{2x} - u_{2x}) = -3\Delta v$$

$$|u_{1x} - v_{1x}| = 3\Delta v$$

Problem 3: (24 points)

A block of mass m_1 is placed on an inclined plane. It is connected to the block of mass m_2 by an unstretchable string of negligible mass. (Note that this problem will not be graded without the Free Body Diagrams).



a) Assume that the surface of the inclined plane is frictionless, and the pulley is frictionless and massless. At what value(s) of θ will the block of mass m_1

i) Move up the ramp at constant speed

$$\begin{cases} -m_1 g \sin\theta + T = m_1 a_x = 0 \\ m_2 g - T = m_2 a_y = 0 \end{cases}$$

$$T = m_2 g; \quad m_1 g \sin\theta = m_2 g$$

$$\boxed{\sin\theta = \frac{m_2}{m_1}}$$

ii) Accelerate up the ramp

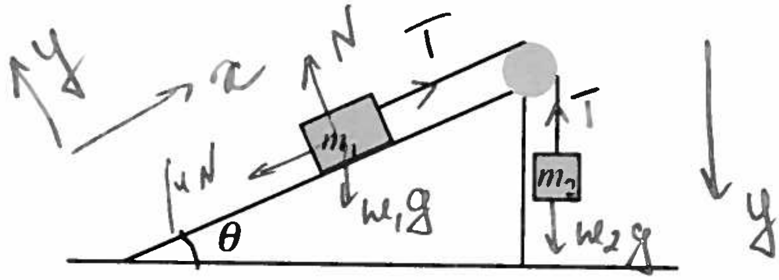
$$\begin{cases} -m_1 g \sin\theta + T = m_1 a_x \\ m_2 g - T = m_2 a_x \end{cases}$$

$$-m_1 g \sin\theta + m_2 g = (m_1 + m_2) a_x$$

$$a_x = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$$

$$m_2 g - m_1 g \sin\theta > 0 \qquad \boxed{\sin\theta < \frac{m_2}{m_1}}$$

b) If the coefficient of friction between the block of mass m_1 and the incline is μ, find value of θ at which the block of mass m_1 will move up the ramp at constant speed. Assume that the pulley is frictionless and massless. Write the equation, do not solve it.

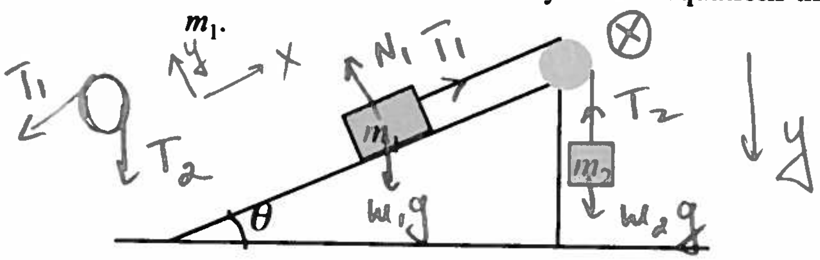


$$\begin{cases} T - \mu N - m_1 g \sin\theta = 0 \\ N = m_1 g \cos\theta \\ m_2 g - T = 0 \end{cases}$$

$$m_2 g - \mu m_1 g \cos\theta - m_1 g \sin\theta =$$

$$\boxed{m_2 - \mu m_1 \cos\theta - m_1 \sin\theta = 0}$$

c) Assume that the surface of the inclined plane of a given angle θ is frictionless, but the pulley has radius R and moment of inertia I with respect to the axis of rotation. The string pulls without slipping. In the box below write the system of equations that can be solved to find the acceleration of the block m_1.



$$T_1 - m_1 g \sin\theta = m_1 a_x \quad (1)$$

$$m_2 g - T_2 = m_2 a_x \quad (2)$$

$$R T_2 - R T_1 = I \alpha$$

$$a_x = R\alpha$$

$$R(T_2 - T_1) = I \frac{a_x}{R}$$

$$(1) + (2):\ T_1 - m_1 g \sin\theta + m_2 g - T_2 = (m_1 + m_2) a_x$$

$$-I \frac{a_x}{R^2} - m_1 g \sin\theta + m_2 g = (m_1 + m_2) a_x$$

d) Solve for acceleration of the block m_1.

$$\boxed{a_x = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2 + \frac{I}{R^2}}}$$

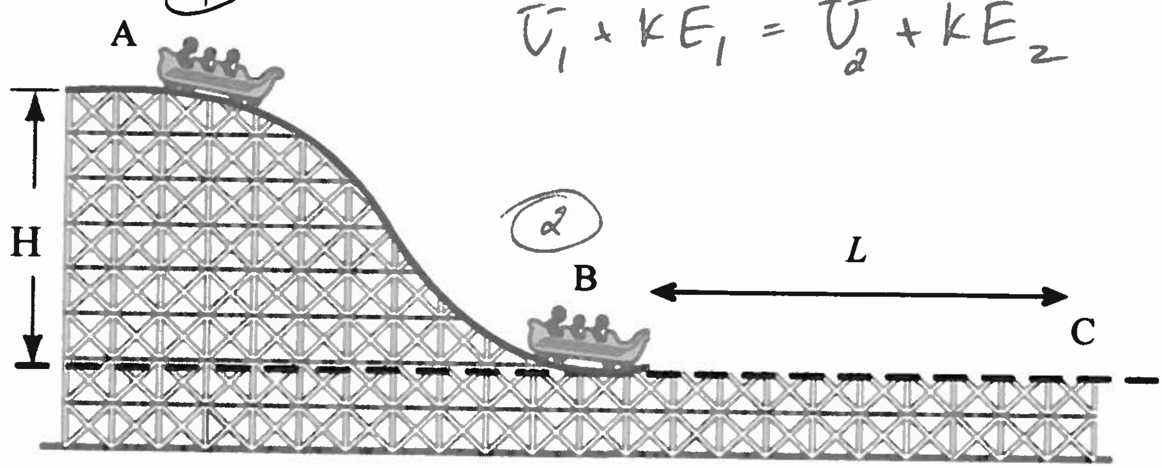
Problem 4: (16 points)

A roller coaster of mass *m* starts at rest at height *H* and falls down the path from point A to point B without friction. (Ignore the rotational motion of the wheels).

a) Find the speed of the car at point B.

1

$$U_1 + KE_1 = U_2 + KE_2$$



$$mgH = \frac{mv^2}{2}$$

$$v = \sqrt{2gH}$$

b) The horizontal surface from B to C is not frictionless. The coefficient of friction between the car and the horizontal surface is

$$\mu = \mu_0(1 + \frac{x}{c})$$

where μ_0 and c are known constants. Obtain the equation that can be solved to find the distance *L* where the car will stop. Do not solve it.

$$W^{net} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$$

$$-\int_0^L \mu_0\left(1 + \frac{x}{c}\right) mg\, dx = 0 - \frac{m\cancel{2}gH}{\cancel{2}} = -mgH$$

$$-\mu_0 mg\left(x + \frac{x^2}{2c}\right)\Big|_0^L = -mgH$$

$$\boxed{\mu_0\left(L + \frac{L^2}{2c}\right) = H}$$

Problem 5: (15 points)

A platform can rotate, without friction, about a vertical axle through its center. A bug of mass m stands at the edge of a platform of radius R and moment of inertia I about the axle. The system is set spinning with an angular velocity ω_0 in the clockwise direction (see Fig 1).)
a) If the bug crawls in toward the center of the platform and stops at a distance b from the center (see Fig. 2), find the angular velocity of the platform.

$\vec{L}_i = \vec{L}_f$

View from above

Fig. 1

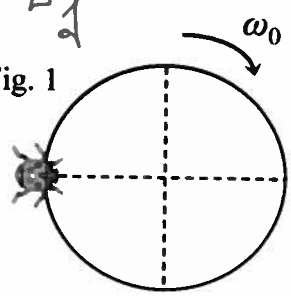


Fig. 2

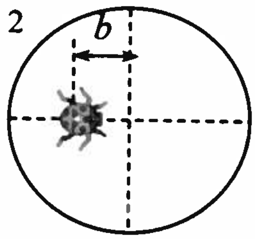
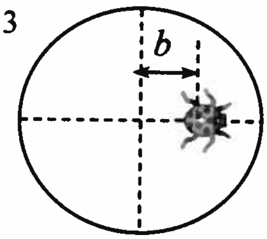


Fig. 3



$$I\omega_0 + mR^2\omega_0 = I\omega + mb^2\omega$$

$$\omega = \frac{(I + mR^2)\,\omega_0}{I + mb^2}$$

b) Instead, the bug stops at a distance b from the center on the other side of the platform (see Fig 3). Find the angular velocity of the platform.

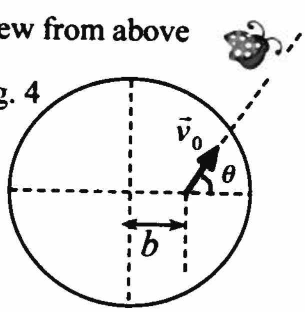
$$I\omega_0 + mR^2\omega_0 = I\omega + mb^2\omega$$

$$\omega = \frac{I + mR^2}{I + mb^2}\,\omega_0 \quad (\text{same as part a})$$

c) What will be the angular velocity of the platform if the bug flies off the platform with a velocity v_0 in the horizontal plane at an angle θ to the radius (see Fig. 4).

View from above

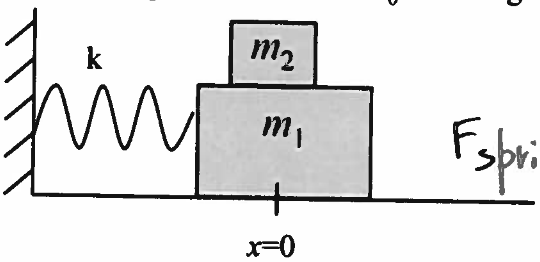
Fig. 4

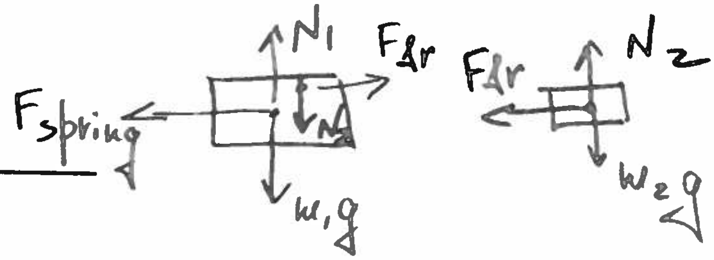


$$mR^2\omega_0 + I\omega_0 = -mv_0 b\sin\theta + I\omega$$

$$\omega = \frac{(mR^2 + I)\,\omega_0 + mv_0 b\sin\theta}{I}$$

Problem 6 (20 points).
A spring with constant k is attached to a block of mass m_1 on a frictionless table. A block of mass m_2 is placed on the top of block m_1. The coefficient of static friction between the two blocks is μ. The blocks are pulled a distance x_0 to the right and released from rest.





a) Draw the Free Body Diagram for both blocks at the initial time moment.

b) Assuming that the blocks move together, find the position of the blocks at any time.

$$-kx = (m_1+m_2)a_x = (m_1+m_2)\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m_1+m_2}x = 0$$

$$x(t) = A\cos\omega t + B\sin\omega t$$

$$x(t=0) = A = x_0$$

$$\frac{dx}{dt} = -A\omega\sin\omega t + B\omega\cos\omega t$$

$$\frac{dx}{dt}(t=0) = B\omega = 0$$

$$B = 0$$

$$\boxed{\omega = \sqrt{\frac{k}{m_1+m_2}}}$$

$$\boxed{x(t) = x_0\cos\omega t}$$

$$\frac{d^2x}{dt^2} = -x_0\omega^2\cos\omega t = a_x(t)$$

$$a_x^{max} = -x_0\omega^2$$

c) (3 points) Find the maximum distance x_0 for which the block m_2 does not slip on the block m_1.

$$-F_{fr} = m_2 a_x$$

$$-\mu N_2 = m_2 a_x^{max}$$

$$-\mu m_2 g = m_2 a_x^{max}$$

$$a_x^{max} = -\mu g$$

$$-\mu g = -x_0\frac{k}{m_1+m_2}$$

$$\boxed{x_0 = \frac{\mu g}{\omega^2} = \frac{\mu g(m_1+m_2)}{k}}$$

Or,

$$\mu N_2 - F_{spring} = m_1 a_{max}$$

$$\mu m_2 g - kx_0 = -m_1\mu g$$

$$-kx_0 = -\mu g(m_1+m_2)$$

$$x_0 = \frac{\mu g(m_1+m_2)}{k}$$