

PHYSICS 218 Final Exam

Fall, 2014

Name: Solutions

Signature: _____

E-mail: _____

Section Number: _____

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- No calculators are allowed in the test.
 - Be sure to put a box around your final answers and clearly indicate your work to your grader.
 - **All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.**
 - Clearly erase any unwanted marks. No credit will be given if we can't figure out which answer you are choosing, or which answer you want us to consider.
 - Partial credit can be given only if your work is clearly explained and labeled. **Partial credit will be given if you explain which law you use for solving the problem.**

Put your initials here after reading the above instructions:

For grader use only:

Problem 1 (14) _____

Problem 2 (14) _____

Problem 3 (17) _____

Problem 4 (16) _____

Problem 5 (15) _____

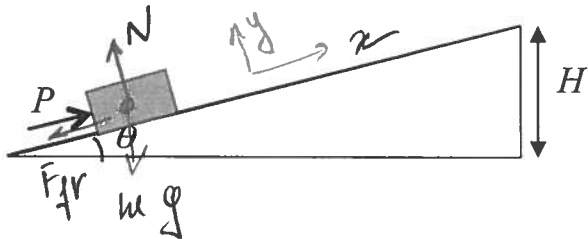
Problem 6 (15) _____

Problem 7 (14) _____

Total (105) _____

Problem 1: (14 points)

A sled of mass m is pushed up the hill by a force P of *unknown* magnitude, which varies with the position of the sled. A coefficient of friction between the sled and the surface is μ . The sled starts at rest at the bottom of the hill of height H and angle θ . When it reaches the top of the hill, it has a velocity of magnitude v_1 . Find the work done by force P .



$$W_{\text{net}} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$$

$$W^P + W^{\text{grav}} + W^N + W^{\text{fric}} = \frac{mv_1^2}{2}$$

$$W^{\text{grav}} = -mgH$$

$$W^N = 0 \quad H/\sin\theta$$

$$W^{\text{fric}} = - \int_0^{H/\sin\theta} \mu N dx = -\mu mg \cos\theta \frac{H}{\sin\theta}$$

$$F_y = ma_y$$

$$N - mg \cos\theta = 0$$

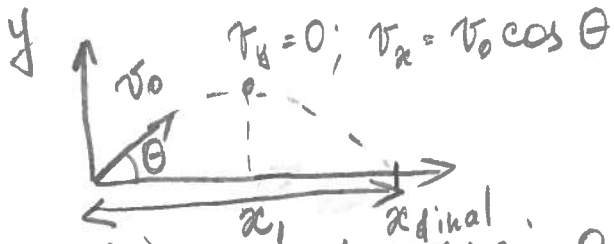
$$N = mg \cos\theta$$

$$W^P = mgH + \mu mg \frac{\cos\theta}{\sin\theta} H + \frac{mv_1^2}{2}$$

Problem 2: (14 points)

A cannonball of mass m is fired at an angle θ above the horizontal and with a speed of v_0 . At the highest point of its trajectory the cannonball explodes into two fragments with equal mass, one of which falls vertically with zero initial speed. Ignore air resistance.

How far from the point of firing does the other fragment strike if the terrain is level?



$$v_y(t) = -gt + v_0 \sin \theta$$

$$y(t) = -\frac{gt^2}{2} + v_0 \sin \theta t$$

highest point: $v_y = 0$ $\boxed{t^* = \frac{v_0 \sin \theta}{g}}$

$$x_1 = x(t^*) = v_0 \cos \theta t^*$$

$$x_1 = \frac{v_0^2 \cos \theta \sin \theta}{g}$$

$$p_x(\text{before}) = p_x(\text{after})$$

$$m v_0 \cos \theta = \frac{m}{2} u_x \Rightarrow \boxed{u_x = 2 v_0 \cos \theta}$$

$$x_{\text{final}} = u_x t^* + x_1 = \frac{3 v_0^2 \cos \theta \sin \theta}{g}$$

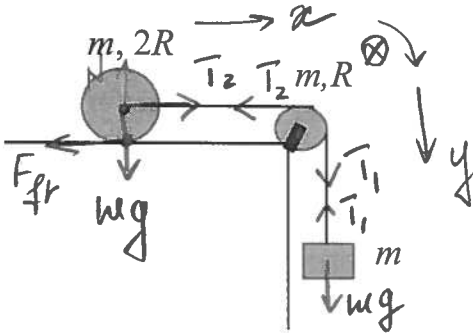
Answer:

$$\boxed{\frac{3 v_0^2 \sin \theta \cos \theta}{g}}$$

Problem 4: (16 points)

A uniform solid cylinder with mass m and radius $2R$ rests on a horizontal tabletop. A string is attached to a frictionless axle through the center of the cylinder so that the cylinder can rotate about the axle. The string runs over a disk-shaped pulley mass m and radius R that is mounted on a frictionless axle through its center. A block of mass m is suspended from the free end of the string. The string does not slip over the pulley, and the cylinder rolls without slipping on the table top. Note: the moment of inertia for a cylinder of mass m and radius r is $I = mr^2/2$.

a) In the box below write the system of equations that can be solved to find the acceleration of the block and the force of friction between the cylinder and the surface after the system is released from rest. The problem will not be graded without the free body diagrams and coordinate systems.



$$\begin{aligned} mg - T_1 &= ma \quad (1) \\ T_2 - F_{fr} &= ma \quad (2) \\ T_1 R - T_2 R &= \frac{mR^2}{2} \alpha \quad (3) \\ a &= R\alpha \quad (4) \\ F_{fr} 2R &= \frac{m(2R)^2}{2} \alpha_1 \quad (5) \\ a &= 2R\alpha_1 \quad (6) \end{aligned}$$

b) Solve for the acceleration of the block and the force of friction between the cylinder and the surface.

$$\begin{cases} mg - T_1 + T_2 - F_{fr} = 2ma \\ (T_1 - T_2)R = \frac{mR^2}{2} \frac{a}{R} \\ F_{fr} 2R = \frac{m}{2} 4R^2 \frac{a}{2R} \end{cases}$$

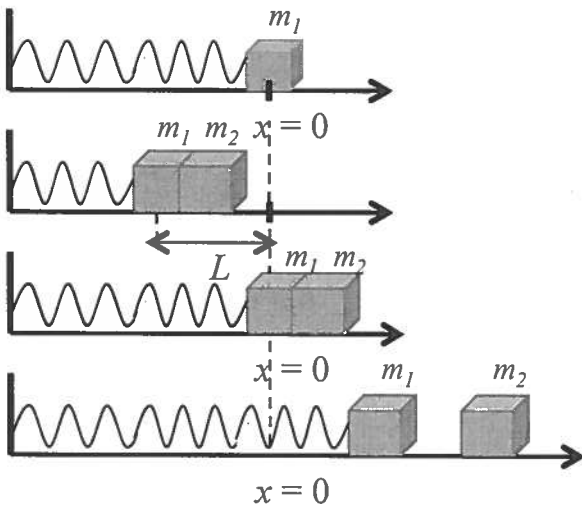
$$\begin{cases} mg - \frac{ma}{2} - F_{fr} = 2ma \\ F_{fr} = \frac{ma}{2} \end{cases}$$

$$\begin{aligned} mg - \frac{ma}{2} - \frac{ma}{2} &= 2ma \\ 3ma &= mg \\ \boxed{a} &= \frac{g}{3} \end{aligned}$$

$$\begin{aligned} \text{Answer: } a &= g/3 \\ F_{fr} &= mg/6 \end{aligned}$$

Problem 3 (17 points)

A block of mass m_1 is connected to a light spring of constant k that is attached to a wall. A second block, mass m_2 is pushed against m_1 compressing the spring by the amount L . The system is then released, and both objects start moving to the right on the frictionless surface. When m_1 reaches the equilibrium point, m_2 loses contact with m_1 and moves to the right with velocity v .



a) Determine v , velocity of m_2 .

$$\frac{kL^2}{2} = (m_1 + m_2) \frac{v^2}{2}$$

$$v = \sqrt{\frac{k}{m_1 + m_2}} L$$

b) Find the position of the block m_1 as a function of time if we started the clock when m_1 was at $x=0$ just as m_2 left?

$$v(t=0) = \sqrt{\frac{k}{m_1 + m_2}} L$$

$$x(t=0) = 0$$

$$-kx = m_1 \frac{d^2 x}{dt^2}$$

$$\frac{d^2 x}{dt^2} + \frac{k}{m_1} x = 0$$

$$x(t) = A \cos \omega t + B \sin \omega t$$

$$x(t=0) = A = 0$$

$$\frac{dx}{dt}(t=0) = B\omega = \sqrt{\frac{k}{m_1 + m_2}} L$$

$$B = \sqrt{\frac{k}{m_1 + m_2}} \frac{L}{\omega}$$

$$-B\omega^2 \sin \omega t + \frac{k}{m_1} B \sin \omega t = 0$$

$$\omega = \sqrt{\frac{k}{m_1}}$$

$$x(t) = L \sqrt{\frac{m_1}{m_1 + m_2}} \sin \sqrt{\frac{k}{m_1}} t$$

c) How long will it take for the block m_1 to come to the point where the spring has its maximum extension?

$$v = 0$$

$$\frac{dx}{dt} = B\omega \cos \omega t = 0$$

$$\omega t^* = \frac{\pi}{2}; \quad t^* = \frac{\pi}{2\omega} = \frac{\pi \sqrt{m_1}}{2\sqrt{k}}$$

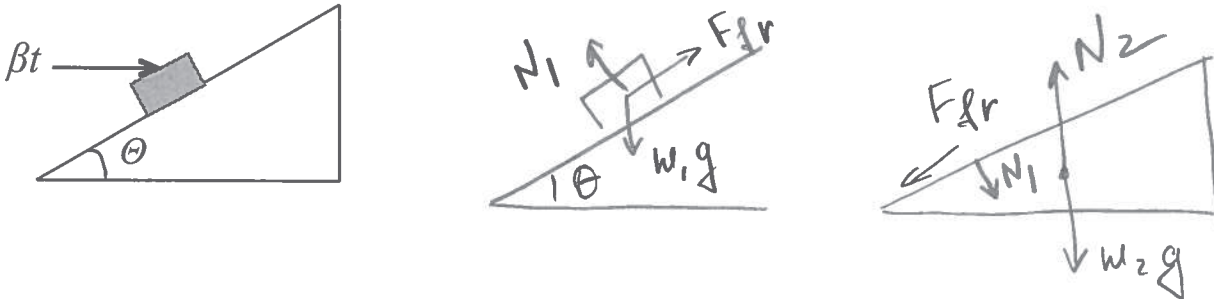
OR

$$T = \frac{2\pi}{\omega}$$

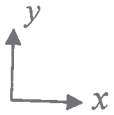
Problem 5: (15 points)

A block of mass m_1 rests on top of a triangular block of mass m_2 , which has angle θ (see the picture below). The blocks are on a frictionless table. The coefficient of friction between the blocks is μ . At time $t=0$ a horizontal force $F(t) = \beta t$, where β is a known constant, is applied to the top block as shown.

a) Draw the free-body diagram for both blocks at $t = 0$.



b) Find the acceleration of the blocks when they move together without slipping. Use the given coordinate system.



$$\beta t = (m_1 + m_2) a$$

$$a = \frac{\beta t}{m_1 + m_2}$$

Answer:

$$a_2 = \frac{\beta t}{m_1 + m_2}$$

c) In the box below write the system of equations that can be solved to find the time when the top block will start slipping the other block. Use the given coordinate system.



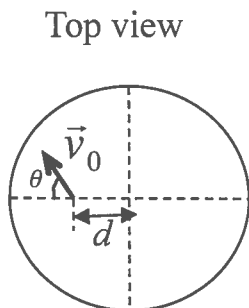
$$\beta t \cos \theta - m_1 g \sin \theta - \mu N_1 = \frac{\beta t m_1 \cos \theta}{m_1 + m_2}$$

$$N_1 - m_1 g \cos \theta - \beta t \sin \theta = -\frac{\beta t m_1 \sin \theta}{m_1 + m_2}$$

Problem 6: (15 points)

A cannon of mass M is firmly mounted on a rotating horizontal platform, radius R , at a distance d from its axis. The platform can rotate with negligible friction about a vertical axis. The moment of inertia of the platform about the vertical axle through its center is a known constant I . The platform is initially at rest. At time $t = 0$ the cannon fires a shell of mass m with initial velocity v_0 directed at an angle α with respect to the horizontal. The angle between the horizontal projection of the velocity and the radius is θ as shown on the picture below.

a) Find the magnitude and direction of the angular velocity of the platform after the shot.



$$L_z(\text{before}) = L_z(\text{after})$$

$$0 = -m v_0 \cos \alpha d \sin \theta + I \omega + M d^2 \omega$$

$$\omega = \frac{m v_0 \cos \alpha d \sin \theta}{I + M d^2} \text{ counterclockwise}$$

b) Immediately after the shot someone applied a brake to the edge of the platform which created a known force of friction F . Find the torque created by force F about the axis.

$$\vec{\tau} = FR \otimes$$

c) How long will it take for the platform to come to a stop after the shell is fired?

$$\vec{\tau} = (I + M d^2) \alpha \quad (\text{rhr})$$

$$-FR = (I + M d^2) \alpha$$

$$\alpha = -\frac{FR}{I + M d^2}$$

$$\omega(t) = \int \alpha(t) dt$$

$$\omega(t) = -\frac{FR}{I + M d^2} t + \omega_0$$

$$\omega_0 = \frac{m v_0 \sin \theta \cos \alpha d}{I + M d^2}$$

$$\omega = 0 \quad t^* = \frac{\omega_0 (I + M d^2)}{FR}$$

$$t^* = \frac{m v_0 d \sin \theta \cos \alpha}{FR}$$

Problem 7: (14 points)

A particle of mass m is attracted to a center by a single force that has only radial component of magnitude

$$|\vec{F}| = \frac{\beta}{r^2}$$

where r is a distance from the center.

a) Find the potential energy function for this force.

$$F_r = -\frac{\beta}{r^2} ; F_r = -\frac{dU}{dr}$$

$$U = -\int F_r dr = -\int -\frac{\beta}{r^2} dr = -\frac{\beta}{r} + \text{Const}$$

b) If the particle was initially located at distance r_0 from the center, what should be its minimum speed in order for the particle just to make it to infinity?

$$\bar{U} + KE = \text{Const}$$

$$-\frac{\beta}{r_0} + \frac{mv^2}{2} = 0$$

$$v = \sqrt{\frac{2\beta}{mr_0}}$$