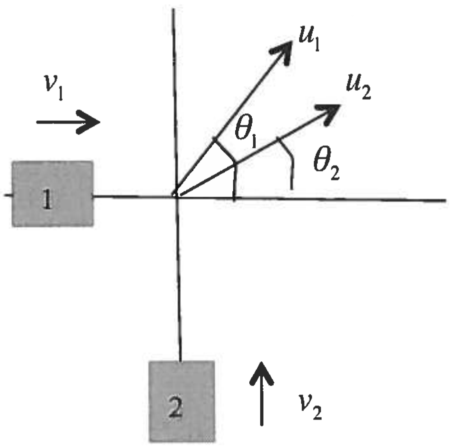
Problem 1: (15 points)

From the skid marks a highway patrolman knows the final directions of two cars which have collided. The collision happened on a slippery road so you can ignore friction. Assuming that the masses of the cars, m_1 and m_2, as well as velocities u_1 and u_2, and angles θ_1 and θ_2 are given, in the box below write the system of equations that could be solved to find the initial velocities of the cars, v_1 and v_2. Solve for v_1 and v_2.



p_x (before) = p_x (after)

p_y (before) = p_y (after)

$$m_1 v_1 = m_1 u_1 \cos\theta_1 + m_2 u_2 \cos\theta_2$$

$$m_2 v_2 = m_1 u_1 \sin\theta_1 + m_2 u_2 \sin\theta_2$$

$$v_1 = u_1 \cos\theta_1 + \frac{m_2}{m_1} u_2 \cos\theta_2$$

$$v_2 = \frac{m_1}{m_2} u_1 \sin\theta_1 + u_2 \sin\theta_2$$

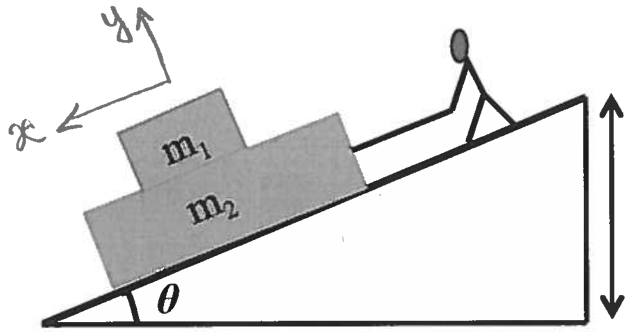
Problem 2: (15 points)

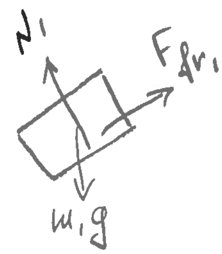
You are lowering two boxes, masses m_1 and m_2, one on the top of the other, down the ramp by pulling on a rope parallel to the surface of the ramp. The coefficient of friction between the blocks is μ_1, and the coefficient of friction between the lower block and the ramp is μ_2. The pulling force of magnitude P is such that they are moving with constant velocity. The angle θ is known.

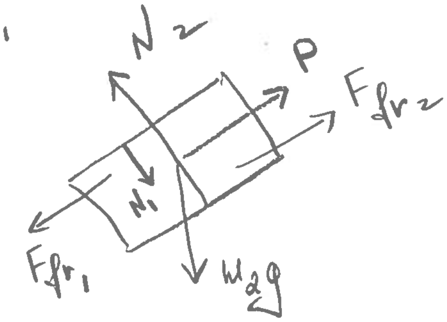
a) Draw the free body diagram for both blocks.

b) In the box below write the system of equations that could be soled to find force P and the force of friction between the two blocks.

c) Find the magnitude of the force P and magnitude of the force of friction between the blocks.







$$\begin{cases} F_x = ma_x \\ F_y = ma_y \end{cases}$$

$$mg\sin\theta - F_{fr_1} = 0$$
$$N_1 - m_1 g\cos\theta = 0$$
$$-P + m_2 g\sin\theta + F_{fr_1} - \mu_2 N_2 = 0$$
$$N_2 - N_1 - m_2 g\cos\theta = 0$$

$$N_1 = m_1 g\cos\theta$$
$$N_2 = (m_1 + m_2) g\cos\theta$$

$$F_{fr_1} = m_1 g\sin\theta$$
$$P = m_2 g\sin\theta + m_1 g\sin\theta - \mu_2 (m_1 + m_2) g\cos\theta$$

Answer: $F_{fr} = m_1 g\sin\theta$

$P = (m_1 + m_2) g\sin\theta - \mu_2 (m_1 + m_2) g\cos\theta$

Problem 3: (15 points)

A satellite of mass m is attracted to the Earth, mass m_e, with a force of magnitude

$$|\vec{F}| = G\frac{m_e m}{r^2}$$

where G is the gravitational constant. Find the potential energy function for this force. Find the work done by this force when the satellite is moved from the orbit of radius R_1 to the further orbit of radius R_2.

$$F_r = -\frac{\partial U}{\partial r} \; ; \quad F_r = -\frac{G m_e m}{r^2}$$

$$U = -\int F_r\, dr = -\int -\frac{G m_e m}{r^2}\, dr =$$

$$= -\frac{G m_e m}{r} + \text{Const}$$

$$W = \int_{r_1,\theta_1}^{r_2,\theta_2} F_r\, dr + \int_{r_1,\theta_1}^{r_2,\theta_2} \overset{=0}{F_\theta}\, r\, d\theta = \int_{R_1}^{R_2} -\frac{G m_e m}{r^2}\, dr =$$

$$= \frac{G m_e m}{r}\Big|_{R_1}^{R_2} = G m_e m\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

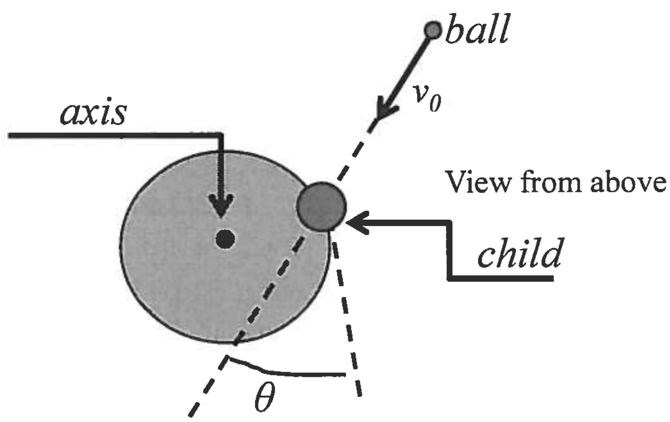
Or

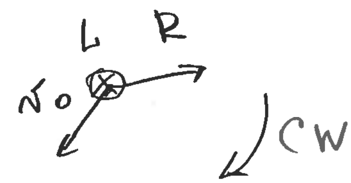
$$W = -[U_2 - U_1] = -\left[-\frac{G m_e m}{R_2} + \frac{G m_e m}{R_1}\right] =$$

$$= G m_e m\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

Problem 4 : (15 points)

A child of mass m_1 stands on the edge of a merry-go-round of radius R. The merry-go-round rotates freely and has moment of inertia I about the axis through its center. The child catches a ball of mass m_2 thrown by a friend. The ball has a horizontal velocity of magnitude v_0 at angle θ with a line tangent to the outer edge of the merry-go-around, as shown. What is the angular velocity (magnitude and direction) of the merry-go-round just after the ball is caught?





$$\vec{L}_i = \vec{L}_f$$

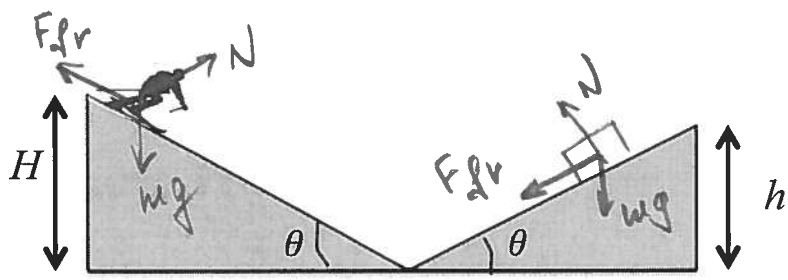
$$m_2 v_0 \cos\theta R = I\omega + (m_1 + m_2) R^2 \omega$$

$$\omega = \frac{m_2 v_0 \cos\theta R}{I + (m_1 + m_2) R^2} \quad \text{CW}$$

Problem 5: (15 points)

Two snowy peaks are at heights H and h above the valley between them. A ski run extends between the peaks at an average slope of θ.

a) A skier starts from rest at the top of the higher peak. At what speed will he arrive at the top of the lower peak if he coasts without using ski poles? Ignore friction.



$$U_1 + KE_1 = U_2 + KE_2$$

$$mgH = mgh + \frac{mv^2}{2}$$

$$\boxed{v = \sqrt{2g(H-h)}}$$

b) What coefficient of friction between the snow and skis would make him stop just at the top of the lower peak?

$$W^{net} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

$$W^{grav} = mgH - mgh$$

$$W^{N} = 0$$

$$F_y = ma_y$$

$$N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

$$W^{fric} = \int_0^{\frac{H}{\sin\theta}} -\mu N\,dx + \int_0^{\frac{h}{\sin\theta}} -\mu N\,dx =$$

$$= -\mu mg\cos\theta\frac{H}{\sin\theta} - \mu mg\cos\theta\frac{h}{\sin\theta} = -\mu mg\frac{\cos\theta}{\sin\theta}(H+h)$$

$$mg(H-h) - \mu mg\frac{\cos\theta}{\sin\theta}(H+h) = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = 0$$

$$\mu = \frac{H-h}{H+h}\tan\theta$$

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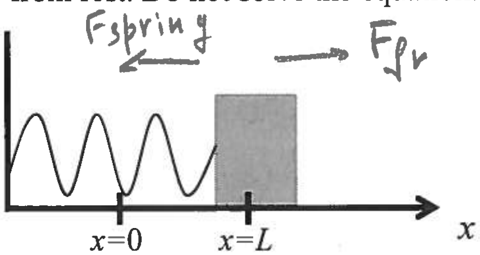
Problem 6: (20 points)

A block of mass m is attached to a spring, spring constant k, and is pulled a distance L from the point at which the spring is unstretched. The force of spring is a returning force $F_x = -(kx - bx^3)$ The coefficient of friction between the block and the surface is μ.

a) Find the equation that could be solved to find x_s, the point at which the block will stop if released from rest. Do not solve the equation.



$$W_{net} = \frac{m v_2^2}{2} - \frac{m v_1^2}{2}$$

$$W_{fric} = \int_L^{x_s} \mu N\, dx = \mu m g (x_s - A)$$

$$W_{spring} = \int_L^{x_s} -(kx - bx^3)\, dx = -\frac{kx^2}{2} + \frac{bx^4}{4} \Big|_L^{x_s} = -\frac{kx_s^2}{2} + \frac{bx_s^4}{4} + \frac{kL^2}{2} - \frac{bL^4}{4}$$

$$-\frac{kx_s^2}{2} + \frac{bx_s^4}{4} + \frac{kL^2}{2} - \frac{bL^4}{4} + \mu m g (x_s - L) = 0$$

b) Put $b = 0$. Ignore friction ($\mu = 0$). The block is pulled a distance L from $x = 0$ and released from rest. Derive the equation that describes the position of the block as a function of time. How long will take the block to return to the unstretched position?

$$-kx = m \frac{d^2x}{dt^2} \qquad x(t=0) = L \qquad v(t=0) = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{k}{m} x = 0}$$

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$x(t=0) = A = L$$

$$\frac{dx}{dt} = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$

$$\frac{dx}{dt}(t=0) = B\omega = 0 \Rightarrow B = 0$$

$$-A\omega^2\cos(\omega t) + \frac{k}{m} A\cos(\omega t) = 0$$

$$\omega^2 = \frac{k}{m}$$

$$\boxed{x(t) = L\cos\left(\sqrt{\frac{k}{m}}\, t\right)}$$

$$t^* = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{2\pi\sqrt{m}}{4\sqrt{k}} = \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

Or $x(t^*) = 0$

$$\cos\sqrt{\frac{k}{m}}\, t^* = 0$$

$$\sqrt{\frac{k}{m}}\, t^* = \frac{\pi}{2}$$

$$\boxed{t^* = \frac{\pi}{2}\sqrt{\frac{m}{k}}}$$

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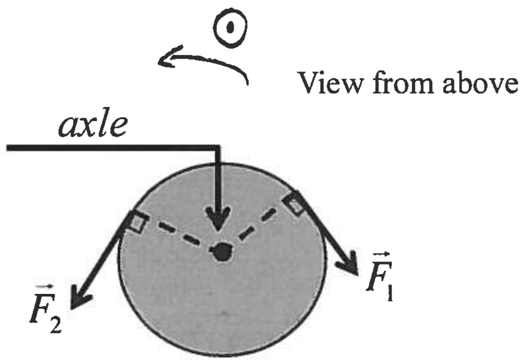
Problem 7: (10 points)

A disk with moment of inertia I about its axle is initially at rest. Starting at time $t = 0$, two constant forces are applied tangentially to the rim as shown, so that at time t_1 the disk has an angular velocity of ω_0 counterclockwise. Magnitude of force F_1 is a known quantity. What is magnitude of F_2? R is given.



$$\omega_0 = \alpha t_1$$

$$\alpha = \frac{\omega_0}{t_1}$$

$$\vec{\tau}_{ext} = \underline{I}\,\alpha \;(\text{rhr})$$

$$RF_2 - RF_1 = \underline{I}\,\frac{\omega_0}{t_1}$$

$$\boxed{F_2 = \frac{1}{R}\left(\underline{I}\,\frac{\omega_0}{t_1} + RF_1\right)}$$